LEIBNIZ AND MANY-VALUED LOGICS

Prof. Dr. Ángel Garrido / Prof. Dr. Piedad Yuste

Faculty of Sciences / Faculty of Philosophy

UNED (Universidad Nacional de Educación a Distancia)

Paseo Senda del Rey, 7, 9, 28040 – Madrid, Spain

agarrido@mat.uned.es / pyuste@fsof.uned.es

In memory of Prof. Quintin Racionero.

Abstract. Our initial purpose is contributing to the search for the origins of many-valued logics (MVL, by acronym), its possible relation with the philosophical tradition derived from G. W. Leibniz, and, within MVLs, as a special case that of "Fuzzy Logic". Because Leibniz can be considered as the true precursor of modern Formal Logic and this is due to two main issues: The first one would be to develop a logical `ideography´, or `characteristica universalis´; the second reason would be given by his research on the so-called `racionator calculus´. For all these reasons it is a remarkable forerunner of modern mathematical logic. It is also our goal to relate how was welcome to Leibniz, Lukasiewicz, Zadeh, and modern logics in our Iberian peninsula, which are just another province of the world philosophical universe.

1. INTRODUCTION

Searching for the origins of MVL could lead too far and eventually disperse, which, as we know is not very convenient for a job pretending to be research. So, we may analyze the problem of "future contingents", treated by Aristotle in *Peri hermeneias.*

About Future Contingent Propositions, we must remember that they are statements about states of affairs in the future that are neither necessarily true nor necessarily false. Suppose that a sea-battle will not be fought tomorrow. Then it was also true yesterday (and the week before, and last year) that it will not be fought, since any true statement about what will be the case was also true in the past. But all past truths are now necessary truths; therefore it is now necessarily true that the battle will not be fought, and thus the statement that it will be fought is necessarily false. Therefore it is not possible that the battle will be fought. In general, if something will not be the case, it is not possible for it to be the case: "For a man may predict an event ten thousand years beforehand, and another may predict the reverse; that which was truly predicted at the moment in the past will of necessity take place in the fullness of time" (Aristotle, *Peri Hermeneias,* ch. 9). The "problem of the future's contingents" is a logical paradox concerning the contingency of a future event. It was first proposed by Diodorus Cronus

(4th century BC) of the Megarian School. Being a problem so-called Master Argument, then reactualized by the Stagirite, because Aristotle admitted that its logical laws did not apply to future events (`Sea Battle Paradox´, on the aforementioned *Peri hermeneias*, Ch. IX): "A sea fight must either take place tomorrow or not, but it is not necessary that it should take place tomorrow, neither is it necessary that it should not take place, yet it is necessary that it either should or should not take place tomorrow". This paradoxical problem gives origin to Modal Logic, and in the hands of Jan Lukasiewicz to the three value logic, in which a third truth value expressing indeterminacy is introduced: "Tout d'abord, en tant que fondateur des systèmes de logique propositionelle multivalents, j'affirme que ces systèmes […] sont nés sur fond de recherches logiques portant sur les propositions modales et les concepts de possibilité et nécessité"[1]. About of this, and according to the Polish philosopher, "If statements about the future events are already true or false, then the future is as much determined as the past and differs from the past only in so far as it has not yet come to pass" [2]. So, the battle-sentence falls in the wide category of future contingent sentences which refer to the future not needed, or not actually determined events. Possibly, Aristotle intuited here the existence of the "third" logical status of the propositions. But he is not interested to develop a theory in the style of what later will be multivalued logics (with three, four and even infinite truth values).

The Stoics were determinists, while the Epicureans were not. Such problems were also discussed by G. W. Leibniz[3], in *Essais de Theodicée sur la bonté de Dieu, la liberté de l'homme, et l'origine du mal* (I, §§36-55 and II, §§169-176), and *Discours de méthaphysique* (§13). His purpose, such as Lukasiewicz more of two centuries later, was to try to reconcile human freedom with divine predestination. However for Leibniz the judgments about the future can only be true or false, leaving no option for one third truth value, because the indetermination has no place in the world created by God and governed by principles of sufficient reason and contradiction. As we all know, Leibniz opened another branch of modal logic: the deontic logic (*Elementa Juris Naturalis*). It is very debatable whether there is evidence of some similar to a three-valued logic in Aristotle (according to future contingent), but the first known 3-valued logic was developed in 1909, by the Russian mathematician and poet Nicolai A. Vasiliev, and his Imaginary Logic. He put forward for the first time ever the idea of (non-Aristotelian) logic, free of the laws of excluded middle and contradiction. Thus, he was a forerunner of paraconsistent and multi-valued logics.

The famous Moravian logician Kurt Gödel (1906-1978), in 1932, showed that the so-called Intuitionistic Logic is not a finitely-many valued logic. Because this, it defines

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[1] Lukasiewicz, *En défense de la logistique*, 2013, p. 302.

[2] It is the leitmotiv of Jan Lukasiewicz, its main intellectual obsession, which leads to create the multi-valued logic, the very classic and difficult problem of determinism. This will connect, in the distance, with Aristotle, and in more recent times, with the great tradition inaugurated by Leibniz, with Bolzano, Brentano, Lukasiewicz, Tarski, but also including Cantor and his Classical Set Theory, now generalized by Zadeh and his Fuzzy Set Theory, etc. Also remember that "Lukasiewicz seems to have been, in practice, a bigger `Bolzanizer´, in the Polish tradition, than Twardowski" (Chrudzimski, 2006, p. 68.) "… il faut apprendre à penser clairement, logiquement et précisémént. Toute la philosophie modern a été handicapée par l´incapacité a penser clairement, précisément et de maniére scientifique" (Luk., *Wyadomosci Literakcie*, 1(19), 1924).

[3] According to Couturat on Leibniz: "sa logique était, non seulement le coeur et l´âme de son systeme, mais le centre de son activité intellectuelle et la source of toutes ses inventions" (Préface a *La Logique de Leibniz*).

an intermediate phase between Intuitionistic Logic and Classical Logic[4]. So, the Aristotelian paradigm was not essentially questioned until the twentieth century, when thinkers such as Bertrand Russell, Max Black, Lofti A. Zadeh, or Jan Lukasiewicz (among others) already had enough theoretical tools available to rescue and interpret the notion of vagueness that Aristotle had not been interested to analyze. And after the seminal papers of the aforementioned Zadeh, from which derives many of the more innovative and interesting new Mathematics, with very useful and good applications.

Jointly with the contribution of Jan Lukasiewicz (1878-1955), also Alfred Tarski (1902-1983) formulated a logic of n truth values, being n > 2, clarifying many important technical questions, by the initial introduction of a third value, the so-called "possible" or "indeterminate" state, denoted by I or ½, and related with the Sea Battle Paradox. Jan Lukasiewicz considers three values, 0, 1 and 2, where the additional value 2 must be the value of future contingent sentences, interpreted as "possibility" or "indeterminacy", while 0 and 1 are the classical values of full trueness and full falseness, respectively. But soon he changed the notation, putting ½ instead of 2, suggesting that the natural 'ordering' of the three values had reflected his intuitions concerning propositional connectives better.

As we known, although the starting point of Leibniz `calculus universalis´ was the original theory of syllogisms of Stagirite, but Leibniz ends for independence from the theories of Aristotle, to finally develop its own axiomatic system, a more general type, based on applying the `combinatorial instrument´ to syllogistic. This is another aspect in which coincides with Lukasiewicz: the Attempt to formulate and axiomatizing the Aristotelian syllogistic, as is developed in Geometry. There are many similarities between both philosophers: the search for a rigorous and univocal universal language through which to express not only the statements of sciences but also the philosophical thoughts. In short: it is necessary create a scientific method for the Science and Philosophy by means the mathematical logic or logistics; *universal characteristic*, according to Leibniz. Lukasiewicz was aware that this discipline was invented by the Saxon thinker[5], and although there were some discrepancies in relation to his metaphysical conception of the universe, the Leibnizian spirit permeated all his works: "Il faut se baser sur des propositions qui sont aussi claires et certaines que possible du point de vue intuitif et adopter ces propositions à titre d'axioms […] Il faut tenter de réduire au maximum le nombre d'axiomes et de concepts primitifs et le compter tous avec soin…"[6]

That issue, of Future Contingent´s problem, with variations, would then central in medieval times, as during the Scholasticism, with William of Ockham, and Duns Scotus, or Richard of Lavenham, among others, looked at from different point of views, for his relationship with determinism and `Divine Foreknowledge´. Then, this issue is taken up by Spanish Jesuit Luis de Molina (and the famous controversy `De Auxiliis´ maintained with the Dominican Domingo Báñez), or the much admired and read Francisco Suarez, also a Jesuit, and even the great polymath G. W. Leibniz dedicated his time[7]. The controversial "De Auxiliis" involves two key works: the *Concordia*, from

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[4] According to R. Consuegra, "Gödel defended a dualistic ontology based on a sort of Leibnizian monadology, an epistemology based on a probable rationalistic knowledge" (2001, p. 6).
[5] Lukasiewicz (2013), *Sur le Déterminisme*, § 1.
[6] Lukasiewicz (2013), *Logistique et Philosophie*, p. 320.
[7] Leibniz distinguishes two types of necessity: *absolute necessity* and *hypothetical necessity*, or universal necessity vs. singular necessity. Universal necessity concern universal truths, while singular necessity

Luis de Molina (1535-1600), and the *Apology*, from Domingo Báñez (1528-1604), from the Dominican School of Salamanca. In essence, it represented the possible antagonism between free will of humans and efficacy of Divine Grace. In short: How is compatible omniscience and omnipotence of God with freedom man? The discussion took a particularly interesting way during the Middle Age. In this period, the logic connected their philosophical discipline with theology. And one of the most important theological issues was precisely the problem of future contingents, in its direct relationship with Christian doctrine. According to this tradition, related with the Divine Foreknowledge. It includes knowledge of future possibilities to be made by beings humans. But this assumption seems to lead to a simple argument. It leads from foreknowledge to the need of future events: now known as God and I will take the decision tomorrow, it´s true that my choice of morning is given. My choice then, it seems necessary but not free. Therefore, there appears to be no basis for claim that we have freedom of choice among alternatives. The conclusion, however, would violate the idea of human freedom and of moral responsibility[8].

Even then there is a dark time for the logic, and reappearing in the nineteenth century, philosophers and mathematicians such as Cantor, De Morgan, Boole, Frege... There was born the new set theory, now called "classic", but then also had terrible enemies, as Leopold Kronecker, who from his professorship in Berlin did everything possible to hinder the work of Cantor[9], and the rise of those new ideas. Another opponent to this new theory was the French mathematician Henri Poincare, who saw them as a disease or a fad. But other great mathematicians, as his friend R. Dedekind or the patriarch of Göttingen, D. Hilbert, were determined defenders of set theory[10].

2. MANY-VALUED LOGICS AND LVOV-WARSAW SCHOOL

Parallel to this, there arises a new kind of thought and way of seeing must be the act of philosophizing: the Polish Lvov-Warsaw School[11] (LWS, by acronym). This is happening like tributaries of a great river and sub-tributaries, departing from Leibniz[12],

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concerns something necessary which could not be; it is, thus, a "contingent necessity" (see *DM*, §13; *Théodiceé,* §37). Thus, to avoid falling into determinism, Leibniz proposes to distinguish between absolute necessity and hypothetical necessity; the latter is reaching contingent events when they occur; therefore, it is possible to assign a truth value to an event that will occur or not in the future, based on the conditions that determine that happening at the present time today.

[8] "Leibniz had defended free will against Spinoza… Leibniz in turn found followers in Bolzano, Herbart and their disciples, who invoked Leibniz in order to refute Kant and his successors." (Johnston, 1983, p. 290.)

[9] Curiously, and according to Agarwal and Sen, "it has been recently discovered that Set Theory had existed long before Christ, at the Jain School of mathematics in India [*Creators…,* pp. 298-301]. This may be an interesting line of future historical research. It was also studied by Singh (1988), and Joseph (2000). It may be considered as precursor the Indian *`doctrine of uncertainty´ (anakantavada),* about the principles of multiplicity of viewpoints and pluralism, fundamental of Jainism. Only the *kavalis* (omniscient beings) can comprehend objects in all aspects. Others only are capable of partial knowledge. Therefore, no human view can claim to represent absolute truth; only different degrees of truth.

[10] Hilbert predicted: "No one will drive us from the paradise which Cantor created for us" (1926, p. 170).

[11] Both the number of representatives as well as the breadth of productions predominated school Lemberg -Warsaw, founded by Twardowski. The anti-irrationalism was its main feature. This school had an analytical orientation. Strongly linked with logistics and especially designed for the clarity of the style of thinking.

[12] "The progenitor of philosophy in Austria… was the Saxon Leibniz, who spent two years in Vienna from 1712 to 1714. It was there that he wrote his *Monadologie,* and *Principes de la Nature et de la Grâce"*. (Johnston, 1983, p. 274.) "Leibniz would be the first modern analytical philosopher, and in fact,

from masters to disciples[13]. Start with the aforementioned Bernard Bolzano[14], which influenced about his intellectual heir, Franz Brentano[15]. This, in turn, greatly influence on all his subsequent students. Among these disciples of Brentano will be one that particularly interested us. This was the Polish philosopher Kazimierz Twardowski, who shared many characteristics with his teacher: love for precision and clarity of ideas, charisma among those who treated him, preference for the spoken to the written word, etc… From his chair in the city of Lvov (Lemberg) spread many of the ideas of Brentano, adding their own. Led to a circle of people, all they very interested in a compulsory renewal of philosophical studies, especially from the point of view of logic. In a certain sense, served a function similar (but independent) to the Vienna Circle (Wiener Kreis), or later the Berlin Circle, because are very different and singular their characteristics. It´s called Lvov-Warsaw School. It was during the "interbellum", or period between the two World Wars, i.e. ranging from 1918-1939. Then, rouse the Diaspora, after the war and by the strong communist dictatorship.

Many notable names among the members of this school of logic, but could cite to Jan Lukasiewicz, Stanislaw Lesniewski, Kazimierz Ajdukiewicz, Tadeusz Kotarbinski, Mordechai Wajsberg, Alfred Tarski, Jerzy Slupecki... or Andrzej Mostowski. Also must be cited Jan Wolenski (as vindicator of the LWS´ memory), or Helena Rasiowa, Roman Sikorski, etc. Among them, one of the most interesting must be Jan Lukasiewicz, the father of many-valued logics (MVLs, by acronym). Lukasiewicz began teaching at the University of Lwow, and then at Warsaw, but after World War II must to continue in Dublin[16].

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we can trace a subterranean current joining Leibniz to Bolzano… Bolzano teaching was decisive for the destiny of so-called Austrian philosophy and of mathematics in the XIXth century." (Künne, 1998, p. 34).

[13] Lorenzo Peña (in 1995) proceeds to examine the notion of truth Leibnizian did in the context of connections between logic and ontology in Leibniz´s work; more concretely, in its *Generales Inquisitiones de Analysi Notionum et Veritatum* (Couturat, 1903, pp. 356-399). Truth equates it to Existence, while Falsehood to Non-Existence. This identification between Truth and Existence leads (by continuity) to that they are recognizing and opening to a whole infinite number of degrees of truth.

[14] Recall that according to Husserl, Bolzano was one of the greatest logicians of all time. He was also a continuator, in some aspects, of the Leibnizianism. For his *Theory of Science* (1837) can be considered the second founder of Mathematical Logic. Bolzano´s posthumously published (by his disciple, Franz Prihonsky) work *Paradoxien des Unendlichen* (1851) was greatly admired by many eminent logicians, as Peirce, Cantor, and Dedekind. Bolzano attempted to provide logical foundations for all sciences, avoiding paradoxes, and introduce for first time the term `menge´, denoting the `set´ in the Cantorian sense, having a prominent role the distinction between `parts´ and `wholes´. In more recent times, there have been French philosophers trying to use Leibnizian thoughts (as Michel Serres), even Zermelo-Fraenkel theory with the axiom of choice (ZFC) to support an ontology of mathematical basis, as Alain Badiou. All this shows, in greater or lesser degree, an intense Leibnizian flavor. Not forgetting the foregoing, as the case of the also French Louis Couturat, who not only released much of the Leibnizian legacy and offered his own analysis of it, but one of his works, *Algèbre de la logique*, laid the foundation of the training of those in Poland (members of the LWS during the `interbellum´, 1918-1939) were to lay the foundations for what would be many-valued logics. The same Cantor considers his set theory as an essential part of metaphysics, being also a powerful branch of mathematical logic.

[15] "Innovations by Franz Brentano (1838-1917) both revised and reinforced Leibnizian tradition…, helped to inaugurate several major streams of modern philosophy. Among these, Husserl´s phenomenology, Meinong´s theory of objects, and Ehrenfels´ theory showed affinity with Bolzano." (Johnston, 1983, p. 290).

[16] "Among the characteristic features of the School (LWS) were its serious approach to philosophical studies and teaching of philosophy, dealing with philosophy and propagation of it as an intellectual and moral mission, passion for clarity and precision, as well as exchange of thoughts, cooperation with representatives of other disciplines at home and abroad, and also fruitful collaboration with mathematicians. The School found its own scientific style of philosophizing and met international

At first, Lukasiewicz introduced the three-valued logic and then generalized to the infinite-valued. That possibility modulation can be expressed by a membership function, which is to come all the unit interval [0,1], instead of being reduced to the dichotomy of classical logic: True vs. False, 0 vs 1, White vs Black, etc., allowing the treatment of uncertainty and vagueness, important not only from the theoretical point of view, but also for applications. The deep and far connection from Leibniz to Lukasiewicz[17], and then to Zadeh, through crossing for it not only Leibniz but also Bolzano[18], Brentano, and Twardowski, has its justification by Wolenski (1989), Murawski (2010), and Garrido (2013-2016).

Furthermore, Lukasiewicz was the effective mentor of Tarski, whereas officially it was Lesniewski. His biographers, A. and S. Feferman, state that "along with his contemporary, K. Gödel, he changed the face of logic in the twentieth century, especially through his work on the concept of truth and the theory of models." Tarski had gone to the US to participate in a conference when Nazi troops invaded his native Poland and could not return to it. Over time, he created in California the most powerful logical school of his time; in fact, you can consider continuing the tradition inaugurated by the LWS, outside the continent in ruins (Europe)[19]. Its `Semantic Theory of Truth´ is one of the greatest achievements of the human thinking of all time. But the writings of Lukasiewicz suffered after a long slumber, which took care to leave an Azeri engineer, Zadeh, who had studied in Tehran, then prosecuted studies at MIT, and eventually made landfall as a professor at the University of California, Berkeley. He was the first to see their potential utility, in 1965; firstly, obtaining a generalized version of the classical theory of sets, now denoted by FST, acronym of the so-called "Fuzzy Set Theory", and later its application to logic, creating the "Fuzzy Logic"[20]. Another interesting aspect that we must note is that even in his own age was the only Zadeh proposed methods for the treatment of uncertainty. So, we have the case of Max Black, or Dieter Klaua, that without getting resonance therefore proposed similar ideas. Also the same Bertrand Russell had treated the issue[21]. We must not forget that Zadeh, an engineer, knew Jan

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standards of training, rigor, professionalism and specialization... Another feature of the LWS was a drive for full, precise and simplest solutions of problems. This 'perfectionism' (Łukasiewicz) caused the Warsaw logicians to frequently release results with a delay, at the risk of losing priority. They delighted in formally perfecting systems, simplifying axioms several times". (Wybraniec-Sk., 2009, p. 8, p. 20)

[17] "Leibniz had made several arithmetization attempt of syllogistic, i.e., to find arithmetic translations of the four propositional types of the square of opposition that would make all the valid assertoric moods into truths of arithmetic and all the invalid ones into arithmetic falsities. The last attempt that he made was the only successful one. It was never published until 1903 (Couturat), and in this edition Lukasiewicz came across it" (Marshall, 1973, p. 238).

[18] According to Glock (2008, p. 66), "Bolzano anticipa ainsi de plusieurs decénnies l´arithmétisation du calcul et de ses resultats pour la théorie des nombres et la theorie des ensembles… L´innovation le plus important de la logique formelle de Bolzano fut la méthode de variation (1837, pp. 147-162)… Sa notion de déductibilité anticipe la notion tarskienne de conséquence logique, et sa notion de proposition logiquement analythique anticipe la notion quinienne de vérité logique." "Il y a des réminiscences leibniziennes dans la philosophie de la mathématique de Bolzano (1810)."

[19] "The most outstanding school of logic in the world after the Second World War was that in California … The Californian School of Logic -because of Tarski- was in many ways similar to its Warsaw ancestor." (Wolenski, 1983, p. 142)

[20] Rescher says (*MVL*, p. 12) that "many-valued logic is of particular interest from the philosophical point of view, because of its deep involvement with such philosophical issues as the question of future contingents, the problem of the logic of indeterminacy in quantum theory, and above all because the multiplicity of diverse systems of many-valued logics poses the question of `alternative logics´, and the whole issue of relativism and conventionalism in logic."

[21] The first work of the mentor and then collaborator of Russell, A. N. Whitehead (ANW), was *A Treatise on Universal Algebra*, published in 1893. That book is a `turn of a screw´, a modern advance toward the

Lukasiewicz investigations so explained his colleague and good friend, the brilliant American logician Stephen C. Kleene.

The British logician Russell (1923, p. 86) wrote: “All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life but only to an imagined celestial existence... logic takes us nearer to heaven than other studies”. And Zadeh (1973, p. 245), “As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics”.

One of the most interesting cases in the history of logic and AI is the country of Romania. We have the most landmark in the mathematician Grigore C. Moisil (1906-1973), who introduced Computer Science in the country, and after he left a very brilliant school of researchers from Romania devoted to mathematics and AI, many of them scattered around the world by the `economic diaspora´, after the Communist period. After World War II, Grigore C. Moisil started teaching Mathematical Logic at Iasi and Bucharest, as he understood that the new emerging field of computers would have enormous repercussions for the social fabric of society. He continued working about the ideas of C. E. Shannon on Circuits, and basically some J. Lukasiewicz fundamental advances on MVLs, where eventually derive the Fuzzy Logic.

The *Lukasiewicz-Moisil Algebras (LMA)* was created by Moisil as an algebraic counterpart for the many-valued logics of Lukasiewicz. They are an attempt to give semantic consistency to n-valued logics. This theory has developed to a considerable extent both as an algebraic theory of intrinsic interest and in view of its applications to logic and switching theory. Also worthy of mention is the figure of Solomon Marcus (1925-), an inspired disciple of G. Moisil, because has made great contributions to many fields, as Logic, Analysis, or Linguistics, of which is one of the founder and principal contributors.

Antonio Monteiro (1907-1980), mathematician born in Portuguese Angola, showed that for every monadic Boolean-algebra we can construct a 3-valued Łukasiewicz-algebra, and that any 3-valued Łukasiewicz-algebra is isomorphic to a Łukasiewicz-algebra thus derived from a monadic Boolean-algebra.

Many papers on MVLs currently come from European Universities and very active research groups[22]. This is possible because very remarkable researchers (in particular on Mathematical Fuzzy Logic) have created a solid and consistent basis for these theories.

Leibnizian ideal, to base all the sciences in the logical. Then there came the project of both, which resulted in three volumes, the *Principia Mathematica*, published between 1913 and 1910; as we known, this monumental task was interrupted, without reaching the fourth volume devoted to geometry, due to mental exhaustion of the authors. It is truly a complex work, where mathematics is completely refers to Logic. Remember that early Russell´s reading of Leibniz, in preparation for his lectures on this thinker, given at Cambridge in 1899, were a key step in their philosophical evolution. After years in which he worked with Russell, ANW was progressively changing their view about reality, which he could only be understood and explained by process. Hence the approach should be dynamic rather than static. Everything in the world is moving from the change. So for this `process philosophy´, he did not believe in the existence of absolute truth, but half-truths, or partial truths, that is, degrees of truth.

[22] It can detect a very persistent influence of Leibnizian ideas on contemporary thought. For example, in the French case, we have the thesis of Michel Serres (1990), or the works of Alain Badiou (1988, 2006) which applies to the ontology the Cantorian theory; specifically, the ZFC, and Forcing, which is a technique discovered by Cohen, being very useful for proving consistency and independence results.

Such has been the case for Petr Hájek, from the Charles University (Prague), P. Cintula, J. Pavelka, L. Behounek, or Vilém Novák, from Ostrava. They have powerful research groups, with publications which are among the most valued in this field. In Poland they follow the great tradition of the LWS of logic and mathematics, and with contributions to research the topic of uncertainty through the *Rough Sets*, by Zdislaw Pawlak (1926-2006), and continued by Andrzej Skowron, among others[23].

3. A STUDY OF CASE: ABOUT THE RECEPTION OF MANY-VALUED LOGICS

It must be mentioned, in medieval times, Raymond Lully, and his book *Ars Magna*. One of the first Hispanic scholars giving notice of the new currents was J. D. Garcia Bacca, who in 1936 published his *Introduction to modern logic,* a work praised by I. M. Bochenski and Heinrich Schölz. Later try so eminent teachers, between them A. Deaño (editor by Lukasiewicz´s selected papers), M. Sánchez-Mazas (interpreting the logico-mathematical ideas of Leibniz, as the known "characteristica universalis"), either Q. Racionero (also a great leibnizian), or M. Sacristán (prosecuted in `academia´ due to its Marxist point of views), all them very often clashing against a very conservative and nothing good context to innovative ideas.

The implementation, extension and cultivation of mathematical logic (and more specifically, of non-classical logics) had considerable difficulties in the Iberian peninsula. Nevertheless, it comes the subsequent flowering of Fuzzy Logic in our peninsula. As said Kochen[24], from Princeton, "Gödel stood logic on the map of mathematics. Today, all reputable mathematical department or of worth (so serious) is staffed with a representative in the field of logic. You may not have more than one or two logical, but surely there will be some." The possible cause of all these nonsense is the dichotomy, even confrontation, between what is called Science and Humanities. Let us not forget the known Frege´s phrase, "Every good mathematician is at least half a philosopher, and a very good philosopher is at least half a mathematician".[25] Also remember the words of Leibniz, "Mathematics is the logic of the imagination".[26]

Although I have left for last, a name should not be omitted landmark, from those that appear only from time to time in Spain. We are referring to the Father Pablo Domínguez Prieto (1966-2009), Spanish philosopher and theologian, who wrote the first major book in Spain on the Lvov-Warsaw School, starting for that of his Ph. D. thesis. Such work is the so-called *Indeterminación y Verdad. La polivalencia lógica en la Escuela de Lvov-Varsovia,* and was published in 1995. Pablo can be considered as

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[23] We note that Fuzzy Logic seems to be the most suitable for the interpretation of our world that is dynamic and is full of uncertainties. It may be considered somewhat related to Eastern philosophical principles, such as the Yin and Yang being the opposite of each other, but coexisting, supporting each other, combining to form a whole. Another key detail to modern science is that the mathematics of quantum theory not work according to the binary logic of zeros and ones, but admits overlapping opposite situations. Such is the case of the famous thought experiment of Schrödinger´s cat.

[24] Simon Kochen (1937-) is a Professor of Mathematics at Princeton. It was the author (jointly with John H. Conway) of the known Free Will Theorem, which belongs to Quantum Theory, and it is related with determinism and free will: if the humans have some free will, then elementary particles have their own small share of this commodity. Such result is based on controversial aspects proposed to exhibit internal contradictions in the new Quantum Physics, as the EPR (Einstein, Podolsky & Rosen) or the K-S (Kochen-Specker) paradoxes.

[25] "Every good mathematician is at least half a philosopher, and every good philosopher is at least half a mathematician". We can found a very similar opinion of Leibniz, in a letter to Malebranche (1699). Also may be attributed to Frege.

[26] He says that *"mathematics is the logic of the imagination"* (C 556).

one of the Spanish forerunner in the study of MVLs, from the philosophical point of view and in particular of the great Polish contribution (LWS) to logic and mathematical fields[27].

Another interesting Spanish author who has been reporting these new streams of logic is Prof. Julián Velarde, with its paper "Polyvalent Logic", or his book *Formal Logic,* a volume II belonging to its *History of Logic.* Also of great interest may be his work *Gnoseology of Fuzzy Systems,* which analyzes the deep philosophical connections of these issues.

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[27] "Probably no other country, taking into account the size of its population, has contributed so greatly to the development of mathematical logic and foundations of mathematics as Poland"." (Fränkel/Bar-Hillel, 1958, p. 200.) And H. Scholz said, in 1931, "Warsaw became the main center of logical studies".

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